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f_B^{stat} and μ_π^2 in quasiclassical approximation of sum rules

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Abstract

In the framework of sum rules with a use of quarkonium mass spectrum, evaluated in the quasiclassical approximation, estimates of leptonic constant $f_B^{stat} \simeq 320 \pm 60$ MeV in a static limit and for the average heavy quark momentum squared $\mu_\pi^2 \simeq 0.5 \pm 0.1$ GeV^2 are obtained.

1. In the Heavy Quark Effective Theory [1], used for the description of strong interaction dynamics of heavy quarks, there are some dimensionful parameters, which determine an accuracy of the leading approximation in infinitely heavy quark limit as well as values of power corrections over $\Lambda/m_Q \ll 1$, where Λ is a scale, determining the heavy quark virtuality inside hadrons. Among such parameters in physics of heavy mesons $(Q\bar{q})$ with a single heavy quark, the most important quantities are the difference between masses of meson and heavy quark $\bar{\Lambda} = M(Q\bar{q}) - m_Q$, the leptonic constant of heavy meson f_Q^{stat} in the static limit $m_Q \to \infty$, and the square of heavy quark momentum μ_π^2 inside the meson. Since those values are determined by QCD at large distances, for estimates one uses nonperturbative approaches, among which the most powerful tool is sum rules [2].

As for the $\bar{\Lambda}$ value, its estimates in the framework of QCD sum rules have been obtained in refs.[3, 4, 5], where $\bar{\Lambda} = 0.57 \pm 0.07$ GeV. Moreover, the "optical" sum rule by Voloshin [6] allows one to get the inequality [7]

$$\bar{\Lambda} > 2\delta_1(\rho^2 - \frac{1}{4}) \simeq 0.59 \text{ GeV} , \tag{1}$$

where ρ^2 is the slope of universal Isgur–Wise function [8], and δ_1 is the difference between the masses of the lightest vector S-wave state and P-wave state for $(Q\bar{q})$ system at $m_Q \to \infty$.

Further, estimates of f_B^{stat} in the framework of QCD sum rules and in lattice computations are in agreement with each other and result in [1]

$$f_B^{stat} = 240 \pm 40 \text{ MeV} . \tag{2}$$

The sum rule estimation of average square of the heavy quark momentum inside the meson gives the value [1, 5, 9]

$$\mu_\pi^2 = 0.5 \pm 0.1 \text{ GeV}^2 , \tag{3}$$

and the inequality [7]

$$\mu_\pi^2 > 3\delta_1^2(\rho^2 - \frac{1}{4}) \simeq 0.45 \text{ GeV}^2 . \tag{4}$$

Note, however, that the values of the parameters δ_1 and ρ^2 are presently rather uncertain, so that bounds (1) and (4) are not the most conservative ones. A special discussion of the μ_π^2 value can be found, for instance, in ref.[1], where the role of a field theory analog for the virial theorem is considered.

In the present letter we consider the QCD sum rules with a use of S-wave level mass spectrum, calculated in the quasiclassical approximation, [10, 11, 12] and obtain estimates of the f_B^{stat} and μ_π^2 values, which agree with the results, given above.

2. In recent papers [10, 11, 12] the QCD sum rules for leptonic constants of S-wave levels in the $(Q_1\bar{Q}_2)$ quarkonium have been considered with the use of the state mass spectrum, calculated in the quasiclassical approximation. For the $1S$-level one has got the expression

$$f_{V,P}^2 \cdot M = \frac{16\alpha_s}{\pi}\, \mu_\pi^2\, \mu\, H_{V,P} , \tag{5}$$

where $\mu = m_1 m_2/(m_1+m_2)$ is the reduced mass of quarkonium, $\mu_\pi^2 = 2\mu\langle T\rangle$ is the average square of quark momentum inside the quarkonium with the mass $M \simeq m_1+m_2$. α_s in eq.(5) is evaluated at the scale of average virtuality of the one-gluon exchange between quarks, so $\alpha_s = \alpha_s^V(\sqrt{2}\mu_\pi)$ in the so-called V scheme [13], where $\Lambda_{QCD}^V = e^{5/6}\Lambda_{QCD}^{\overline{\rm MS}}$. The $H_{V,P}$ factor corresponds to the hard gluon correction to the vector and pseudoscalar currents, respectively, [12, 14, 15]

$$H_{V,P} = 1 + \frac{2\alpha_s^H}{\pi}\Big(\frac{m_2-m_1}{m_2+m_1}\ln\frac{m_2}{m_1} - \delta_{V,P}\Big) ,$$

where

$$\delta_V = \frac{8}{3} , \quad \delta_P = 2 ,$$

and α_s^H is estimated at the scale $\mu_H = e^{3/8}m_Q$ in the V-scheme, if $m_Q = m_1 = m_2$ [16]. The $H_{V,P}$ factors for the quark-to-antiquark annihilation currents differ from the hard gluon correction to the quark-to-quark transition currents [14]. Nevertheless, one can obtain the exact results for $H_{V,P}$ from the factors, calculated in ref.[14], by the symbolic substitutions $V \to P$ and $m_1 \to -m_1$ with the absolute value for the logarithm argument [12]. However, this simple rule is not valid for the scales, determining the

coupling constant. For the vector and axial-vector quark-to-quark transition currents, Neubert found [17]

$$\mu_V = \sqrt{m_1 m_2}\exp\left\{\frac{3}{4}\right\}, \quad \mu_A = \sqrt{m_1 m_2}\exp\left\{\frac{2-5f(m_2/m_1)}{8-12f(m_2/m_1)}\right\},$$

with

$$f(z) = \frac{1+z}{1-z}\ln\frac{1}{z} - 2\,.$$

One can see, that at $m_1 = m_2$ one has $\mu_H \neq \mu_{V,A}$.

Note, in the broad region of average distances between quarks: 0.1 fm < r < 1 fm, where the coulomb-like potential of heavy quark is transformed into the linearly rising confining potential, the average kinetic energy $\langle T\rangle$ is a constant value, independent of μ (i.e. flavours), [18, 19]

$$\langle T\rangle = \text{ const.} \tag{6}$$

This leads to that in the mentioned region of distances, the heavy quark potential is close to the logarithmic one [19], and the quantization by the Bohr–Sommerfeld procedure results in

$$\frac{dM_n}{dn} = \frac{2\langle T\rangle}{n}\,. \tag{7}$$

In accordance with eq.(7) and from spectroscopic data on the charmonium and bottomonium [20], one can get the estimate

$$\langle T\rangle = 0.43 \pm 0.01 \text{ GeV}. \tag{8}$$

However, the polynomial interpolation of masses for the excited states in heavy quarkonia and heavy mesons[1] leads to the value

$$\langle T\rangle = 0.38 \pm 0.01 \text{ GeV}, \tag{9}$$

that is closer to the corresponding parameter of the logarithmic potential [19]. Therefore, in the following estimates we use the value

$$\langle T\rangle = 0.40 \pm 0.03 \text{ GeV}. \tag{10}$$

Note, that the approximate flavour-independence of the level spacing in heavy quarkonia is the experimental observation, that can be reformulated in the framework of phenomenological potential models, giving compact formulae for the excitation energies, used as input parameters, fitted in the models. A special simplification of the level spacing expressions appears in the quasiclassical approximation, described above.

[1] $(Q\bar{q})$ masses are in agreement with estimates in potential models.

In the case of a heavy quarkonium $(Q\bar{Q})$ with a hidden flavour one has $4\mu = M$ and

$$\frac{f_{V,P}^2}{M} = \frac{2\alpha_s}{\pi} \langle T\rangle \; H_{V,P} \simeq \text{ const.}, \tag{11}$$

where one can neglect the variation of $\alpha_s H_{V,P}$ value under the heavy quark mass change [12]. Moreover, one has $f_P \simeq f_V$ within the 5% accuracy. Relation (11) is in a good agreement with experimental values of leptonic constants for ψ- and Υ-particles [10, 11].

Since the threshold of the hadronic continuum in the system with two heavy quarks is determined by masses of heavy mesons $(Q_1\bar{q})$ and $(\bar{Q}_2 q)$, one finds [21]

$$\bar{\Lambda} = \langle T\rangle \ln n_{th} \simeq 0.6 \pm 0.1 \text{ GeV}, \tag{12}$$

where n_{th} is the number of S-levels of heavy quarkonium below the threshold of hadronic continuum $(n_{th}(b\bar{b}) = 4)$, so that the estimation error is, in general, due to the variation of n_{th}, $\delta\bar{\Lambda} = \langle T\rangle \delta n_{th}/n_{th} \simeq 0.1$ GeV.

For a heavy meson $(Q\bar{q})$ a motion of the light current quark in a medium of quark-gluonic condensate plays an essential role. Therefore the most consistent consideration of sum rules requires the use of the operator product expansion for quark currents with the account of vacuum expectation values for operators of higher dimensions. However, one can make the reasonable approximation and consider the case, when the condensate influence generally results in the appearance of an effective mass for the light quark. Such constituent quark can be considered as the nonrelativistic object, moving in the potential of static heavy quark[2]. So, the potential quark models are quite successful in the heavy meson spectroscopy (see, for example, ref.[22]). Further, one can consider the phenomenological expressions, where one does not include condensates, since the latters are implicitly taken into account by means of the introduction of some phenomenological parameters such as the constituent mass.

Within the offered approach, the approximation means that

$$\mu \simeq M(Q\bar{q}) - m_Q = \bar{\Lambda}\ .$$

The introduction of the constituent light quark is the additional, but reasonable assumption to QCD or HQET, of course. It is an analog to a nonperturbative quantity E_c, defining the thershold energy of hadronic continuum in the HQET Laplace sum rules [1, 5]. The E_c value is determined by the stability principle for the calculated parameters such as the leptonic constant, say. The connection of E_c to the quark-meson mass gap is discussed in [1]. The uncertainty of the E_c estimation in HQET is of the same order as that of in the constituent light quark mass. In the finite energy sum rules, consistent with the HQET sum rules, the E_c value is the basic quantity, determining different dimensionful parameters (see ref.[5], where explicit formulae are

[2]This approximation means that the "brown muck" is considered as a whole, i.e. with no internal structure.

given). Thus, the μ value, determined by $\bar{\Lambda}$, has a quite enough accuracy, comparable with the uncertainty in the other approaches.

Then one has

$$\mu_\pi^2 \simeq 2\mu \langle T \rangle \simeq 0.5 \pm 0.1 \text{ GeV}^2 . \tag{13}$$

In ref.[23] one has shown that spectroscopic data on the ψ- and Υ-families give

$$\alpha_s(\psi, \Upsilon) H_{V,P} \Big(\frac{2m_Q}{M}\Big)^2 = 0.21 \pm 0.01 , \tag{14}$$

that is in agreement with the theoretical estimates [12]. Using $\alpha_s^{\overline{\rm MS}}(m_Z) = 0.117 \pm 0005$ [20] as the one-loop value, expressed in the form

$$\alpha_s(m) = \frac{2\pi}{\beta_0(n_f) \ln m/\Lambda^{(n_f)}} , \quad \beta_0(n_f) = 11 - \frac{2}{3} n_f ,$$

one finds $\Lambda^{(5)} = 85 \pm 25$ MeV, so that n_f is the number of quark flavours with $m_{n_f} < m$. We use the one-loop rule for the $\Lambda^{(n_f)}$ determination

$$\Lambda^{(n_f)} = \Lambda^{(n_f+1)} \Big(\frac{m_{n_f+1}}{\Lambda^{(n_f+1)}}\Big)^{2/(3\beta_0(n_f))} ,$$

leading to $\Lambda^{(3)} = 140 \pm 40$ MeV and $\Lambda^{(4)} = 117 \pm 30$ MeV. Further, one takes $m_Q = M(Q\bar{q}) - \bar{\Lambda}$, so that $m_b = m_5 = 4.7 \pm 0.1$ GeV, $m_c = m_4 = 1.4 \pm 0.1$ GeV. One finds

$$\alpha_s^{\overline{\rm MS}}(m_b) = 0.20 \pm 0.02 , \tag{15}$$

that agrees with α_s estimates[3] from experimental values of the leptonic and radiative decay branching fractions for Υ [24] as well as with lattice computations for the $(b\bar{b})$ system spectroscopy [25], where the estimate, close to (15), takes place, too.

Next, the factor of hard gluon correction is equal to

$$H_P = 1.02 \pm 0.01 .$$

It can be represented as the leading order approximation of the renormalization group improved expression[4]

$$H_{V,P}^{RG} = \Big(\frac{\alpha_s(e^{\delta_{V,P}}\mu)}{\alpha_s(m_Q)}\Big)^{4/\beta_0(n_f)} , \tag{16}$$

that is known in HQET [1] at $\delta_{V,P} = 0$. Using eq.(16), one finds

$$H_P^{RG} = 1.008 \pm 0.004 .$$

[3] The recent result by M.Voloshin gives $\alpha_s^{\overline{\rm MS}}(m_b) = 0.185 \pm 0.003$ [16].

[4] Note, that μ is not the renormalization point, as one could think, looking at eq.(16). Hence, the $H_{V,P}$ factors do not contain an explicit renormalization point dependence, which has to cancel against the renormalization point dependence of some other parameters.

So, we use

$$H_P = 1.010 \pm 0.005 \ ,$$

that gives

$$\alpha_s^V(\sqrt{2}\mu_\pi) \ H_P = 0.36 \pm 0.10 \ . \tag{17}$$

Then in accordance with eq.(5) one has

$$f_B^{stat} = 320 \pm 60 \ \mathrm{MeV} \ . \tag{18}$$

3. Thus, in the framework of sum rules with the use of quarkonium spectroscopy, considered in the quasiclassical approximation, one finds the estimates of f_B^{stat} and μ_π^2, which agree with the values, obtained in QCD sum rules for the heavy meson currents.

As one can see, the obtained estimates of f_B^{stat} and μ_π^2 practically are near the bounds, derived in the sum rules [6, 7].

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